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DIOPHANTINE ANALYSIS.

90. Proposed by H. S. VANDIVER, Bala, Pa.

Prove that it is always possible to find an infinite number of positive integral values of x, y, and z, such that the relation $z^2=x^2+bxy+cy^2$ is satisfied, b and c being any integers whatever.

II. Solution by J. H. DRUMMOND, LL. D., Portland, Me.

Take $y=mx$, and $z=nx$ and reduce, and we have $n^2=1+bm+cm^2=$(say) $(pm-1)^2=p^2m^2-2pm+1$, and

$$m=\frac{2p+b}{p^2-c} \text{ and } n=\frac{p^2+bp+c}{p^2-c}.$$

Take $x=p^2-c$, and we have $y=2p+b$ and $z=p^2+bp+c$, in which b and c may be any number and p any number that will make p^2 greater than c.

91. Proposed by LON C. WALKER. A. M.. Professor of Mathematics. Petaluma High School. Petaluma. Cal.

Find the least three positive integral numbers whose sum, sum of their squares, and sum of their cubes shall each be rational squares.

Solution by the PROPOSER.

Let the required numbers by ax, bx, and cx, respectively. Then, by the conditions of the problem, we have

$$(a+b+c)x=\square \ldots.(1), \quad (a^2+b^2+c^2)x^2=\square \ldots.(2), \text{ or}$$

$$a^2+b^2+c^2=\square \ldots.(3); \text{ and } (a^3+b^3+c^3)x^3=\square \ldots.(4), \text{ or}$$

$$(a^3+b^3+c^2)x=\square \ldots.(5).$$

Assume $(a+b+c)x=x^2$; then $a+b+c=x\ldots.(6)$.

Now assume $(a^3+b^3+c^3)x=\alpha^2x^2$; then $x=\dfrac{a^3+b^3+c^3}{\alpha^2}\ldots.(7)$.

Equating the values of x in (6) and (7), and clearing of fractions,

$$(a+b+c)\alpha^2=a^3+b^3+c^3\ldots.(8).$$

By assuming $\alpha=a-b-c$, substituting in (8), and reducing, we get

$$\alpha^2+(b+c)\alpha=3bc\ldots.(9).$$

From (9), $\alpha=-\frac{1}{2}(b+c)\pm\frac{1}{2}\sqrt{(b^2+14bc+c^2)}\ldots.(10)$.

Now let $b^2+14bc+c^2=\square=(b-\dfrac{\beta}{\gamma}c)^2$, from which $\dfrac{b}{c}=\dfrac{\beta^2-\gamma^2}{2\beta\gamma+14\gamma^2}$.

Hence put $b=\beta^2-\gamma^2$ and $c=2\beta\gamma+14\gamma^2$; then $\alpha=6\beta\gamma-6\gamma^2$.

By substituting these values in (3), and reducing, we get